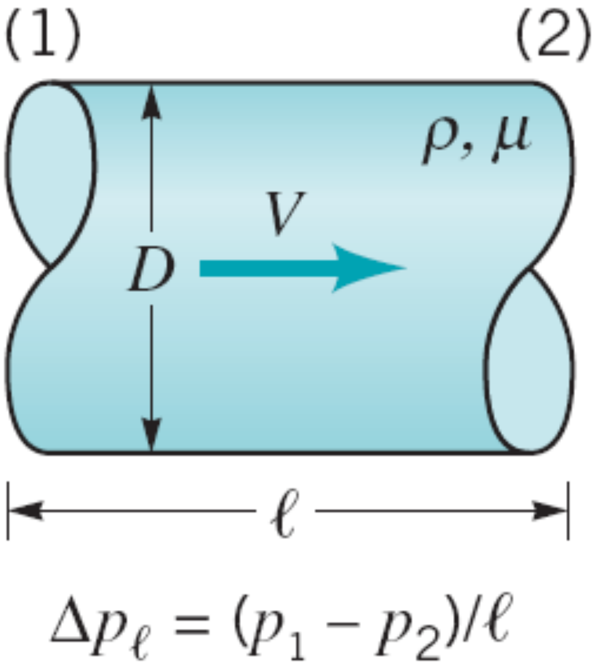
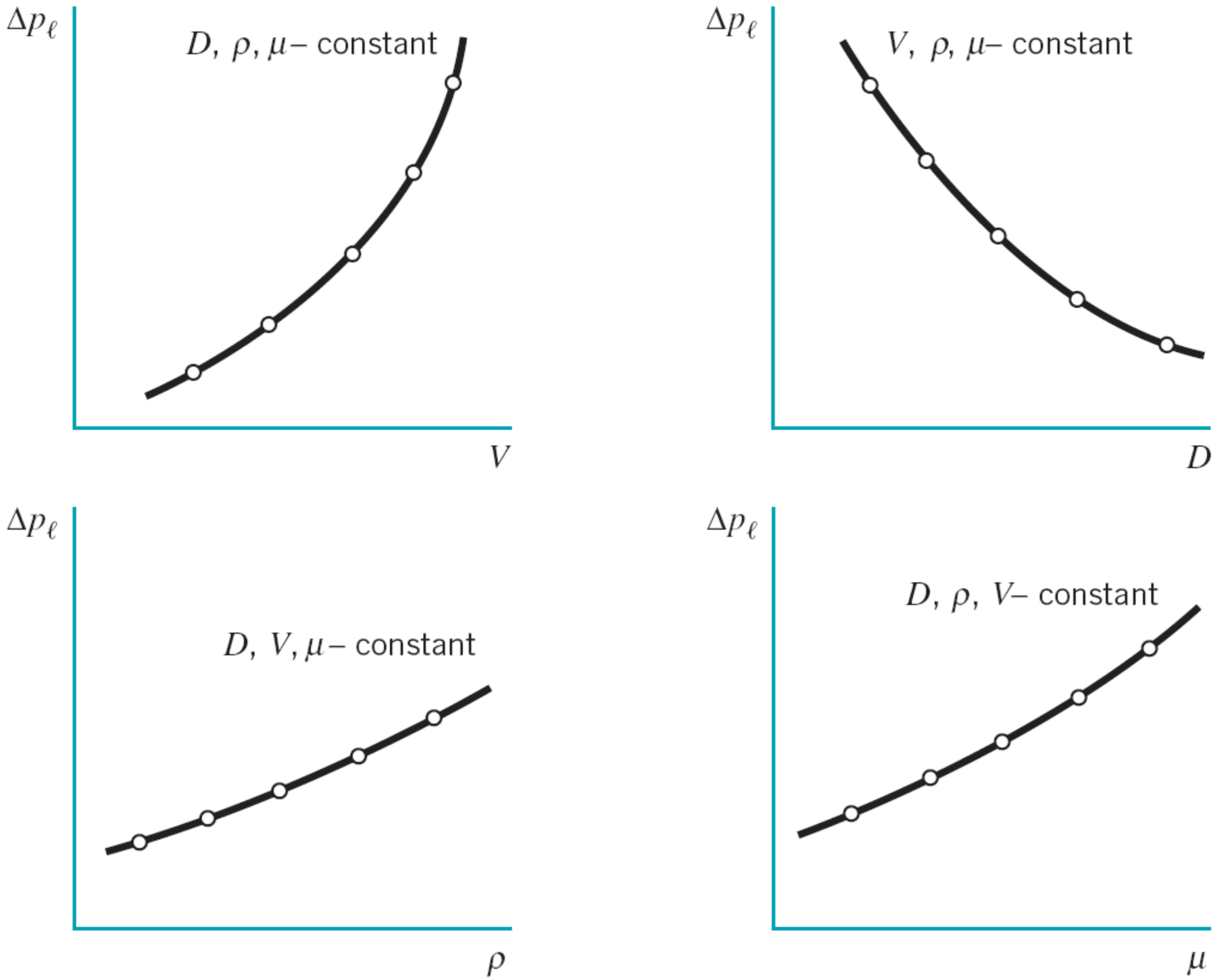
Dimensional analysis

Fluid mechanics is still partly an experimental science



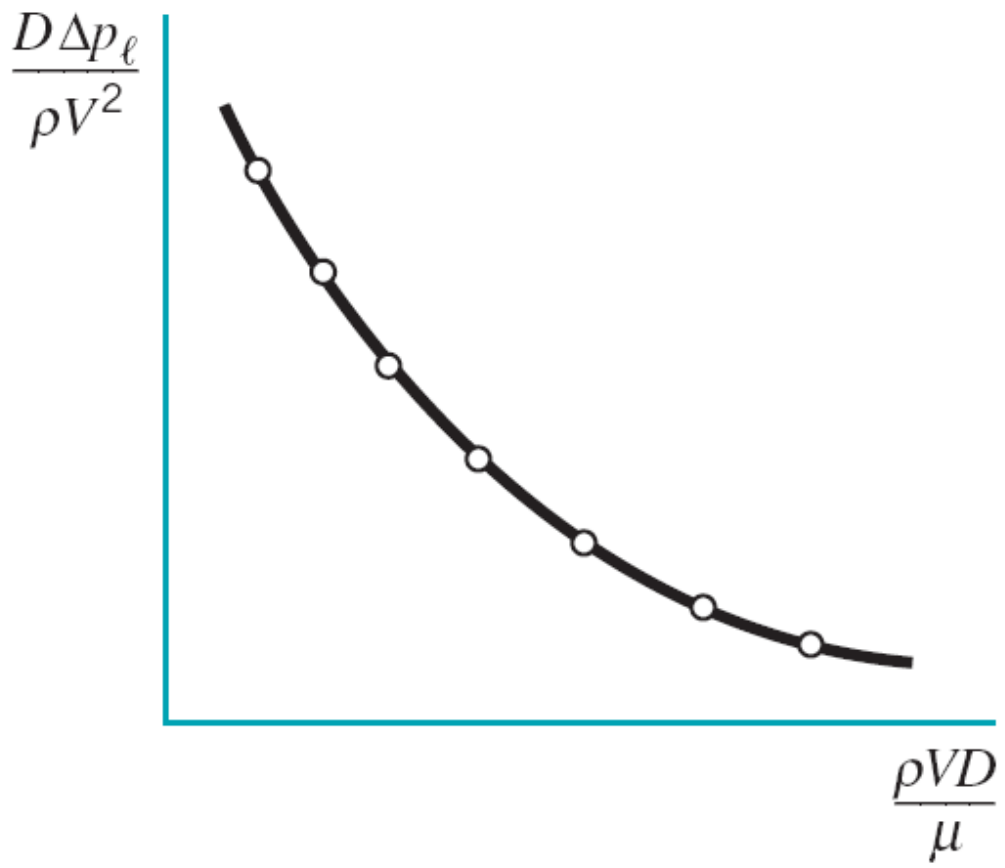
$$\Delta p_l = f(D, \rho, \mu, V)$$



- Expensive
- Impractical (i.e., vary ρ - keep μ constant)
- Difficult to analyse (combine curves)

Best alternative: dimensional analysis

$$\frac{D\Delta p_l}{\rho V^2} = \phi\left(\frac{\rho V D}{\mu}\right)$$



- Only <u>two</u> variables instead of <u>five</u>
- Easier to work with (vary V only, while keeping D, ρ, μ constant)
- Dimensionless groups -> no unit dependance

Buckingham π-theorem

"If an equation involving ***k*** variables is dimensionally homogeneous, it can be reduced to a relationship among ***k – r*** independent dimensionless products (**π-terms**), where ***r*** is the minimum number of reference dimensions required to describe the variables"

Reference dimensions: Usually some or all of **F** (force), **L** (length) and **T** (time)
or **M** (mass), **L** (length) and **T** (time)

or combinations of the basic dimensions (i.e., MT^2, L or FL, T)

Note: We cannot use both F and M, because the two are not independent of each other. Force is related to mass through Newton's 2nd law:

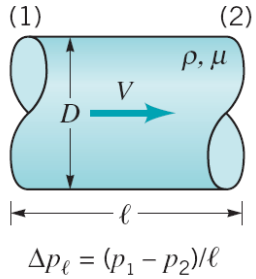
F = M•a

Determination of pi-terms

1. List all variables that are involved in the problem
   - Geometry
   - Fluid properties (density, viscosity, etc.)
   - External effects (velocity, pressure, gravity, etc.)
   - Include variable even if their value is constant (i.e., g)
   - Variables must be independent of each other (i.e., not all of ρ, g amd γbecause $\gamma=\rho$ g)
2. Express each variable in terms of basic dimensions
3. Determine the required number of pi-terms
4. Select a number of repeating variables, where the number required is equal to the number of reference dimensions
5. Form a pi-term by multiplying one of the nonrepeating variables by the product of the repeating variables, each raised to an exponent that will make the combination dimensionless
6. Repeat Step 5 for each of the remaining nonrepeating variables
7. Check all the resulting pi-terms to make sure they are dimensionless
8. Express the final form as a relationship among the pi-terms and think what it means

Example: pipe flow

Step 1



$$\Delta p_l = f(D, \rho, \mu, V)$$

Geometry: D
Fluid properties: μ, ρ
External effects: V

Step 2

$$\Delta p_\ell \doteq (FL^{-2})L^{-1} \doteq FL^{-3}$$

$$D \doteq L$$

$$\rho \doteq ML^{-3} = (FT^2L^{-1})L^{-3} = FL^{-4}T^2$$

$$\mu \doteq FL^{-2}T$$

$$V \doteq LT^{-1}$$

Step 3

5 variables $(\Delta p_\ell, D, \rho, \mu, V)$

$-$ 3 reference dimensions (F, L, T)

$=$ 2 π-terms

Step 4 Select as repeating variables: D, V and ρ

Step 5

$$\Pi_1 = \Delta p_\ell \cdot D^a V^b \rho^c$$

$$\doteq FL^{-3} \cdot (L)^a \cdot (LT^{-1})^b \cdot (FL^{-4}T^2)^c \doteq F^0 L^0 T^0$$

$$\therefore \quad 1 + c = 0 \qquad (F)$$

$$-3 + a + b - 4c = 0 \qquad (L)$$

$$-b + 2c = 0 \qquad (T)$$

$$c = -1 \quad b = -2 \quad a = 1$$

So, $\Pi_1 = \dfrac{\Delta p_\ell \cdot D}{\rho V^2}$

Step 6

$$\Pi_2 = \mu D^a V^b \rho^c$$

$$\doteq (FL^{-2}T) \, L^a (LT^{-1})^b (FL^{-4}T^2)^c = F^0 L^0 T^0$$

$$1 + c = 0 \qquad (F)$$

$$-2 + a + b - 4c = 0 \qquad (L)$$

$$1 - b + 2c = 0 \qquad (T)$$

$$c = -1 \qquad b = -1 \qquad a = -1$$

$$\therefore \quad \Pi_2 = \frac{\mu}{D V \rho}$$

or $$\Pi_2 = \underbrace{\frac{\rho V D}{\mu}}_{\text{Reynolds \#}}$$

Note: We can raise π-terms to any power. They remain valid nondimensional π-terms!

Step 7

Check π-terms:

$$\Pi_1 = \frac{\Delta p_\ell \cdot D}{\rho \cdot V^2} \doteq \frac{FL^{-3} \cdot L}{(FL^{-4}T^2)\cdot(LT^{-1})^2} = F^0 L^0 T^0 \checkmark$$

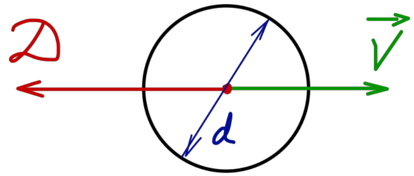
$$\Pi_2 = \frac{\rho V D}{\mu} = \frac{(FL^{-4}T^2)(LT^{-1})L}{FL^{-2}T} = F^0 L^0 T^0 \checkmark$$

Step 8

$$\frac{\Delta p_\ell \cdot D}{\rho V^2} = \phi\left(\frac{\rho V D}{\mu}\right)$$

Example: drag on a sphere

Step 1



$$\mathcal{D} = f(d, \mu, \rho, V)$$

Geometry: d
Fluid properties: μ, ρ
External effects: V

Step 2

$$\mathcal{D} \doteq F$$
$$d \doteq L$$
$$\rho \doteq ML^{-3} = (FT^{2}L^{-1})L^{-3} = FL^{-4}T^{2}$$
$$\mu \doteq FL^{-2}T$$
$$V \doteq LT^{-1}$$

Step 3

5 variables $(\mathcal{D}, d, \rho, \mu, V)$

$-$ 3 reference dimensions (F, L, T)

$=$ 2 π-terms

Step 4 Select as repeating variables: d, V and ρ

Step 5
$$\Pi_1 = \mathcal{D} \cdot d^a V^b \rho^c$$

$$\doteq F \cdot (L)^a \cdot (LT^{-1})^b \cdot (FL^{-4}T^2)^c \doteq F^0 L^0 T^0$$

$$\therefore \quad 1 + c = 0 \qquad (F)$$

$$a + b - 4c = 0 \qquad (L)$$

$$-b + 2c = 0 \qquad (T)$$

$$\Longrightarrow \quad c = -1 \quad b = -2 \quad a = -2$$

So,
$$\Pi_1 = \frac{\mathcal{D}}{\rho V^2 d^2}$$

Step 6
$$\Pi_2 = \mu \, d^a V^b \rho^c$$

$$\doteq (FL^{-2}T) \, L^a (LT^{-1})^b (FL^{-4}T^2)^c = F^0 L^0 T^0$$

$$1 + c = 0 \qquad (F)$$

$$-2 + a + b - 4c = 0 \qquad (L)$$

$$1 - b + 2c = 0 \qquad (T)$$

$$c = -1 \qquad b = -1 \qquad a = -1$$

$$\therefore \quad \Pi_2 = \frac{\mu}{D V \rho}$$

or $\Pi_2 = \dfrac{\rho V D}{\mu}$

Step 7

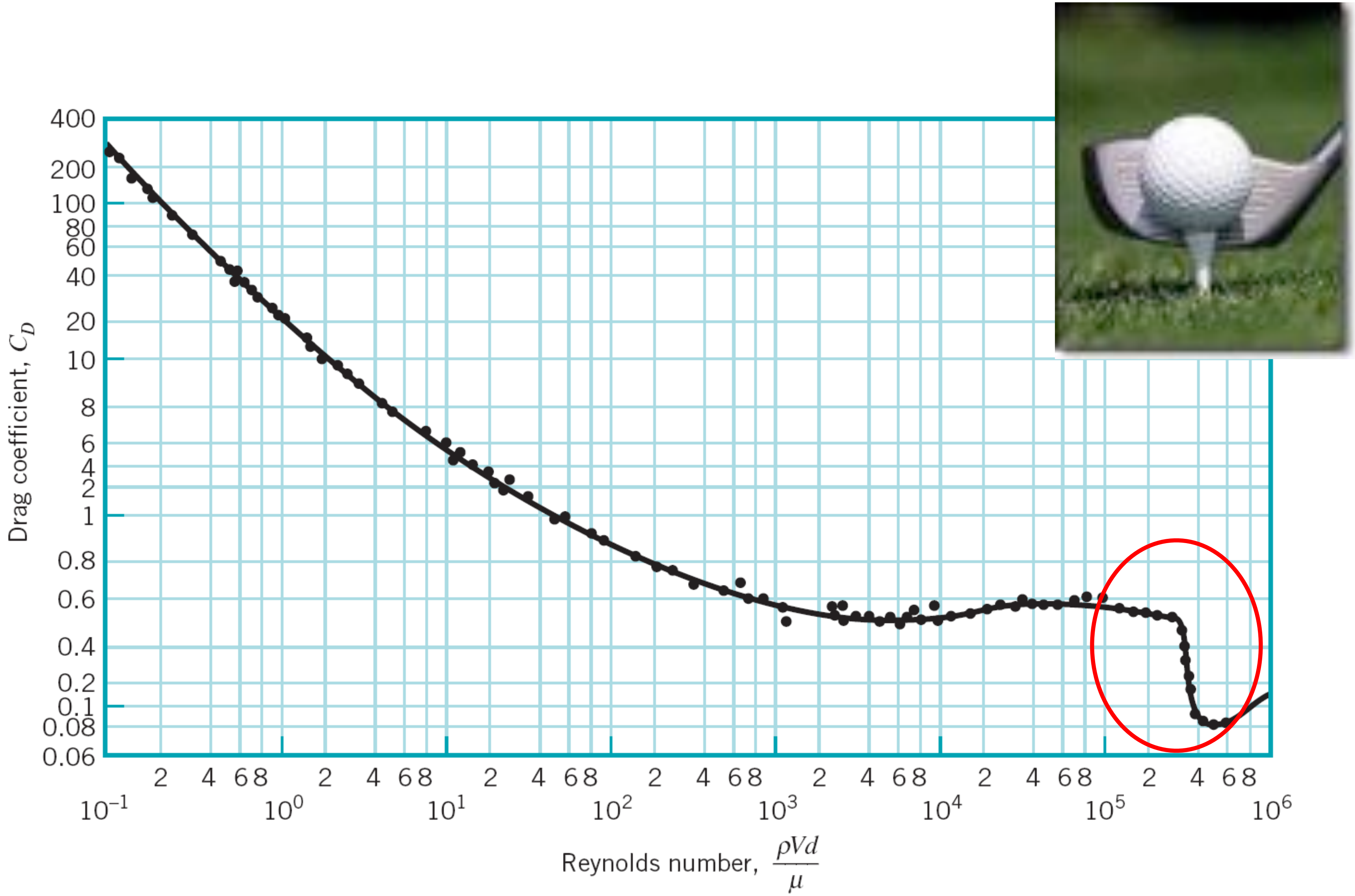
Check Π-terms:

$$\Pi_1 = \frac{\mathcal{D}}{\rho V^2 d^2} \doteq \frac{F}{(FL^{-4}T^2)(LT^{-1})^2 L^2} = F^0 L^0 T^0 \checkmark$$

$$\Pi_2 = \frac{\rho V D}{\mu} = \frac{(FL^{-4}T^2)(LT^{-1})L}{FL^{-2}T} = F^0 L^0 T^0 \checkmark$$

Step 8

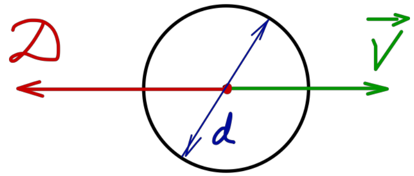
$$\frac{\mathcal{D}}{\rho V^2 d^2} = \phi\left(\frac{\rho V D}{\mu}\right)$$



The effect of Reynolds number on the drag coefficient, C_D for a smooth sphere with $C_D = \mathscr{D}/\tfrac{1}{2} ApV^2$, where A is the projected area of sphere, $\pi d^2/4$

Example: drag on a sphere in viscous fluid (Stoke's law)

Step 1



$$\mathcal{D} = f(d, \mu, V)$$

Geometry: d
Fluid properties: μ
External effects: V

Very viscous fluid or slowly moving sphere. Viscous effects dominate.

Inertia can be neglected

Step 2

$$\mathcal{D} \doteq F$$
$$d \doteq L$$
$$\mu \doteq F L^{-2} T$$
$$V \doteq L T^{-1}$$

Step 3

4 variables $(\mathcal{D}, d, \mu, V)$

$-$ 3 reference dimensions (F, L, T)

$=$ 1 π-term

Step 4 Select as repeating variables: d, V and μ

Step 5
$$\Pi_1 = \mathcal{D} \cdot d^a V^b \mu^c$$
$$\doteq F \cdot (L)^a \cdot (LT^{-1})^b \cdot (FTL^{-2})^c \doteq F^0 L^0 T^0$$

$$\therefore \quad 1 + c = 0 \qquad (F)$$
$$a + b - 2c = 0 \qquad (L)$$
$$-b + c = 0 \qquad (T)$$

$$\Longrightarrow \quad c = -1 \quad b = -1 \quad a = -1$$

So,
$$\Pi_1 = \frac{\mathcal{D}}{dV\mu}$$

Step 6 Not applicable

Step 7 Check π- term:

$$\pi_1 = \frac{\mathcal{D}}{dV\mu} \doteq \frac{F}{L \cdot LT^{-1} \; FTL^{-2}} = F^0 L^0 T^0 \checkmark$$

Step 8

$$\frac{\mathcal{D}}{dV\mu} = ct$$

or

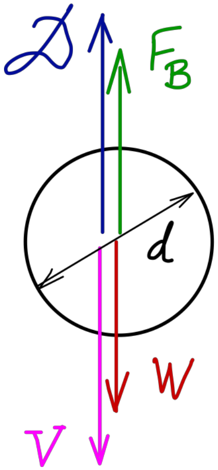
$$\underline{\underline{\mathcal{D} = k \cdot d \cdot V \cdot \mu}}$$

Compare to Stokes' law:

$$\underline{\underline{\mathcal{D} = 3\pi \, d V \mu}}$$

So $k = 3\pi$!!!

For a sphere <u>dropping slowly</u> in a viscous fluid at <u>constant velocity V</u>



$$\sum F_z = 0$$

$$\mathcal{D} + F_B - W = 0 \quad \Rightarrow \quad \mathcal{D} = W - F_B$$

From Stokes' law: $\mathcal{D} \sim dV$

W and F_B are proportional to the volume of the sphere:

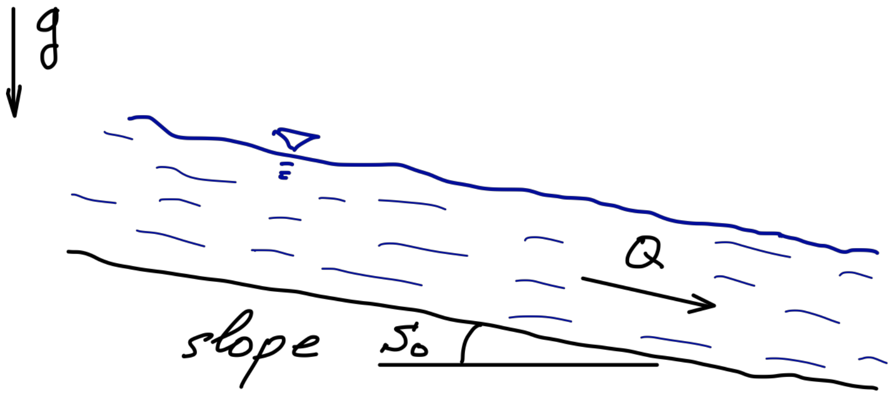
$$W - F_B \sim d^3$$

Hence, $\quad dV \sim d^3$

$$\Rightarrow \underline{\underline{V \sim d^2}}$$

Conclusion: a sphere with double the diameter will fall 4x as fast.

Example: flow in an inclined open channel



$$Q = f(A, \varepsilon, g, S_0)$$

Put the above relation for channel flow Q in **non-dimensional** form.

Can you tell how Q depends on g, A and ε?

A = cross-sectional area of the channel

ε = height of the roughness of the channel surface

Step 1 $\quad Q = f(A, \varepsilon, g, S_0)$

Step 2

$$Q \doteq L^3 T^{-1}$$

$$A \doteq L^2$$

$$\varepsilon \doteq L$$

$$g \doteq L T^{-2}$$

$$S_0 \doteq F^0 L^0 T^0$$

Step 3 5 variables − 2 ref. dimensions = 3 π-terms

Step 4 Select as repeating variables: A and g

Step 5 By inspection:

$$\pi_1 = \frac{Q}{A^{5/4}\sqrt{g}} \qquad \frac{L^3 T^{-1}}{L^{5/2} L^{1/2} T^{-1}} = F^0 L^0 T^0$$

$$\pi_2 = \frac{\varepsilon}{\sqrt{A}} \qquad \frac{L}{L} = F^0 L^0 T^0$$

$$\pi_3 = S_o$$ which is dimensionless

Step 8 $$\frac{Q}{A^{5/4}\sqrt{g}} = \phi\left(\frac{\varepsilon}{\sqrt{A}}, S_o\right)$$

Remarks: a) Q is proportional to $\sqrt{g}$

b) We cannot tell how Q depends on A, ε and S_o because the function ϕ is not known.

TABLE 7.1

Some Common Variables and Dimensionless Groups in Fluid Mechanics

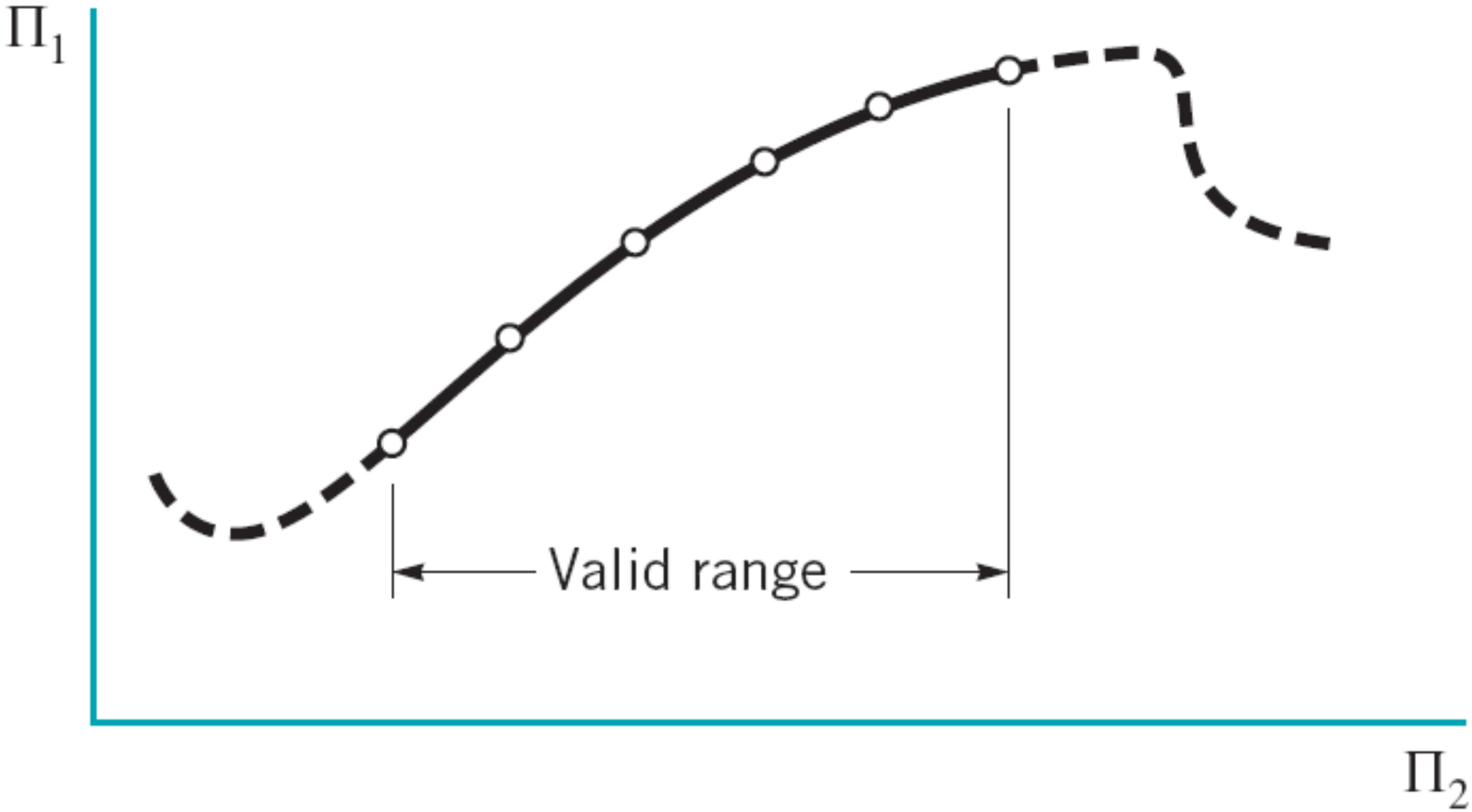
Variables: Acceleration of gravity, g; Bulk modulus, E_v; Characteristic length, ℓ; Density, ρ; Frequency of oscillating flow, ω; Pressure, p (or Δp); Speed of sound, c; Surface tension, σ; Velocity, V; Viscosity, μ

Dimensionless Groups	Name	Interpretation (Index of Force Ratio Indicated)	Types of Applications
$\frac{\rho V \ell}{\mu}$	Reynolds number, Re	$\frac{\text{inertia force}}{\text{viscous force}}$	Generally of importance in all types of fluid dynamics problems
$\frac{V}{\sqrt{g\ell}}$	Froude number, Fr	$\frac{\text{inertia force}}{\text{gravitational force}}$	Flow with a free surface
$\frac{p}{\rho V^2}$	Euler number, Eu	$\frac{\text{pressure force}}{\text{inertia force}}$	Problems in which pressure, or pressure differences, are of interest
$\frac{\rho V^2}{E_v}$	Cauchy number,[a] Ca	$\frac{\text{inertia force}}{\text{compressibility force}}$	Flows in which the compressibility of the fluid is important
$\frac{V}{c}$	Mach number,[a] Ma	$\frac{\text{inertia force}}{\text{compressibility force}}$	Flows in which the compressibility of the fluid is important
$\frac{\omega \ell}{V}$	Strouhal number, St	$\frac{\text{inertia (local) force}}{\text{inertia (convective) force}}$	Unsteady flow with a characteristic frequency of oscillation
$\frac{\rho V^2 \ell}{\sigma}$	Weber number, We	$\frac{\text{inertia force}}{\text{surface tension force}}$	Problems in which surface tension is important

[a]The Cauchy number and the Mach number are related and either can be used as an index of the relative effects of inertia and compressibility. See accompanying discussion.

Problems with 2 π-terms

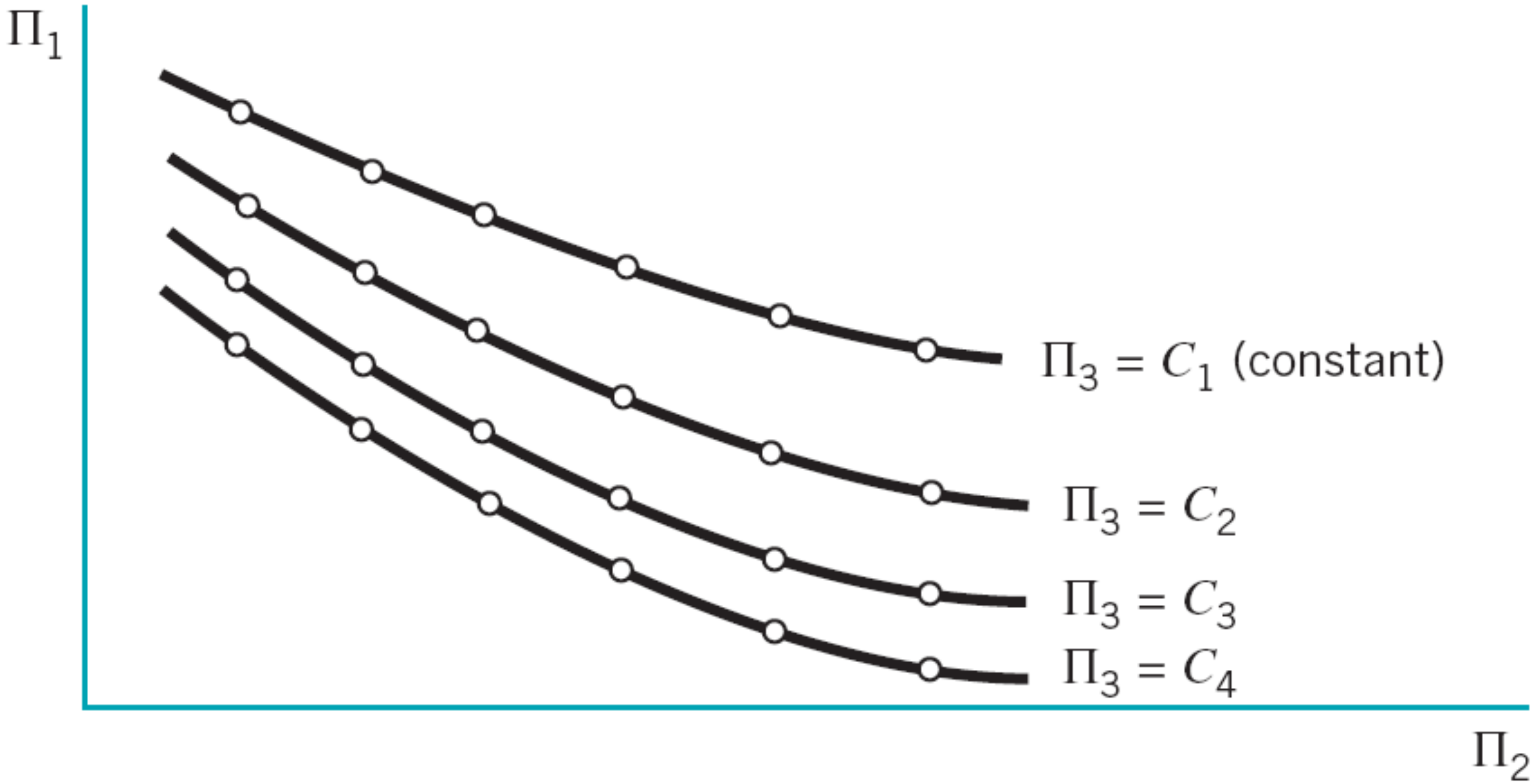
$$\Pi_1 = \Phi(\Pi_2)$$



The graphical presentation of data for problems involving two pi terms, with an illustration of the potential danger of extrapolation of data.

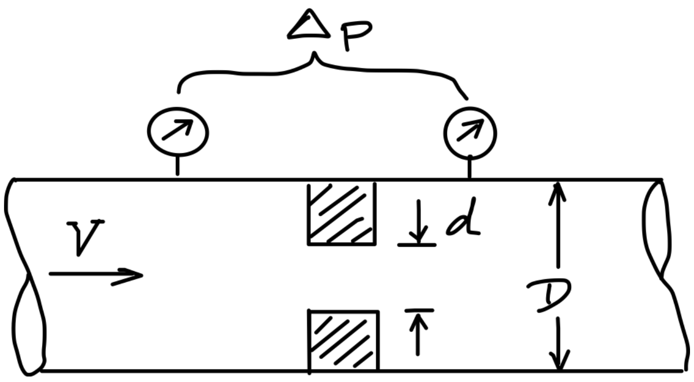
Problems with 3 π-terms

$$\Pi_1 = \Phi(\Pi_2, \Pi_3)$$



The graphical presentation of data for problems involving three pi terms.

Example: pressure drop across a constriction



$V = 0.6\ m/s$

$\rho = 1000\ kg/m^3$

$D = 6\ cm$

Viscous effects not important

Experimental data:

d (cm)	1.8	2.4	3	4.5
Δp (Pa)	22'200	7'030	2'880	570

Derive an expression for Δp.

Step 1 $\quad \Delta p = f(D, d, \rho, V)$

Step 2

$$\Delta p \doteq FL^{-2}$$

$$D \doteq L$$

$$d \doteq L$$

$$\rho \doteq FL^{-4}T^{2}$$

$$V \doteq LT^{-1}$$

Step 3 5 variables − 3 ref. dimensions = 2 π-terms

Step 4 Use d, V and ρ as repeating variables

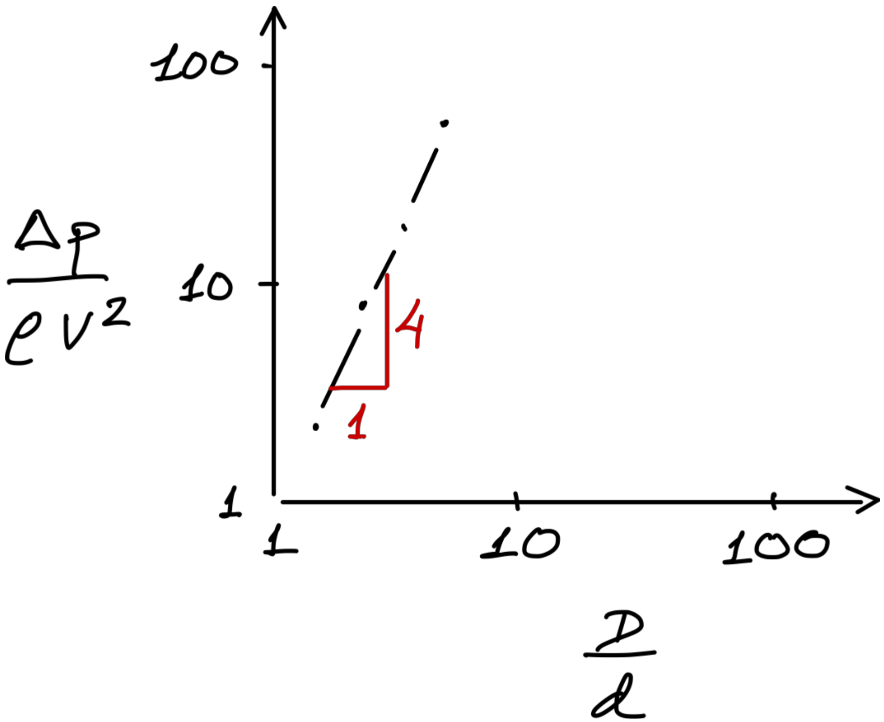
Step 5 $\Pi_1 = \dfrac{\Delta p}{\rho V^2}$

Step 6 $\Pi_2 = \dfrac{D}{d}$

Step 8 $\dfrac{\Delta p}{\rho V^2} = \phi\left(\dfrac{D}{d}\right)$

D/d	3.33	2.5	2.0	1.33
$\Delta p/\rho V^2$	61.7	19.5	8.0	1.58

Obviously <u>nonlinear</u>. Plot on log-log graph



$$\ln\left(\frac{\Delta p}{\rho v^2}\right) = -0.69 + 4 \cdot \ln\left(\frac{D}{d}\right)$$

or $$\frac{\Delta p}{\rho v^2} = 0.5 \cdot \left(\frac{D}{d}\right)^4$$

$$\Rightarrow \boxed{\Delta p = \frac{1}{2}\rho v^2 \left(\frac{D}{d}\right)^4}$$

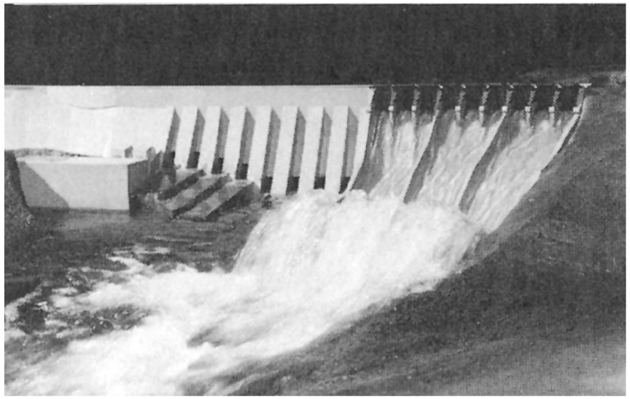
Note: Equation valid only in the range $1.33 < D/d < 3.33$

Modeling

<u>Model</u>: representation of a physical system

$\swarrow$ physical $\qquad \searrow$ mathematical

Use a <u>model</u> to predict the behavior of the <u>actual physical system</u>, which is called the <u>prototype</u>.



An 1:197 scale model of the Guri Dam in Venezuela

Theory of models

For a certain physical phenomenon and for the **prototype:**

$$\Pi_1 = \Phi\left(\Pi_2, \Pi_3, \ldots, \Pi_n\right)$$

For the <u>same</u> physical phenomenon and for the **model:**

$$\Pi_{1m} = \Phi\left(\Pi_{2m}, \Pi_{3m}, \ldots, \Pi_{nm}\right)$$

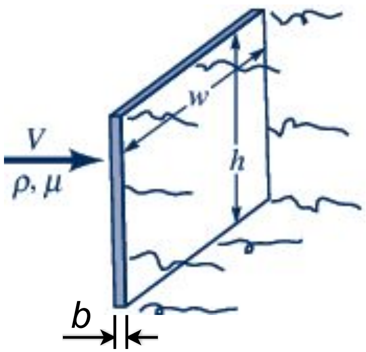
If Φ is the same and

$$\left.\begin{aligned} \Pi_2 &= \Pi_{2m} \\ \Pi_3 &= \Pi_{3m} \\ &\cdots \\ \Pi_n &= \Pi_{nm} \end{aligned}\right\}$$

Design conditions
or
Similarity requirements

then $\Pi_1 = \Pi_{1m}$ Prediction equation

Example: drag on a thin plate



$$D = f\left(w, h, b, \mu, \rho, V\right)$$

Pi-theorem: $$\frac{D}{w^2\rho V^2} = \phi\left(\frac{h}{w}, \frac{b}{w}, \frac{\rho V w}{\mu}\right)$$

Design conditions:

$$\left.\begin{array}{l} \dfrac{h_m}{w_m} = \dfrac{h}{w} \\ \dfrac{b_m}{w_m} = \dfrac{b}{w} \end{array}\right\}$$ geometric similarity

$$\frac{\rho_m V_m w_m}{\mu_m} = \frac{\rho V w}{\mu}$$ dynamic similarity

Note: if $\frac{w_m}{w}$ is selected (i.e., 1:20)

Then $h_m = \frac{w_m}{w} h$

and $b_m = \frac{w_m}{w} b$

so <u>all length scales are the same</u> $\frac{w_m}{w} = \frac{h_m}{h} = \frac{b_m}{b} = n_l$

n_l is the length scale (i.e., 1:20, 1: 1000, etc.)

Can define also velocity scale $n_v = \dfrac{V_m}{V}$

density scale $n_\rho = \dfrac{\rho_m}{\rho}$ etc.

Usually we specify the <u>length scale</u> n_ℓ

<u>Note:</u> The other scales <u>are not</u> in general the same as the length scale

Example: The velocity scale in the plate drag problem is:

$$\frac{\rho_m V_m W_m}{\mu_m} = \frac{\rho V W}{\mu} \Rightarrow \frac{V_m}{V} = \left(\frac{\rho}{\rho_m}\right)\cdot\left(\frac{W}{W_m}\right)\cdot\left(\frac{\mu_m}{\mu}\right)$$

If the same fluid (air) is used in the model $(\rho_m = \rho\,;\ \mu_m = \mu)$, then $\dfrac{V_m}{V} = \dfrac{W}{W_m} = \dfrac{20}{1}$

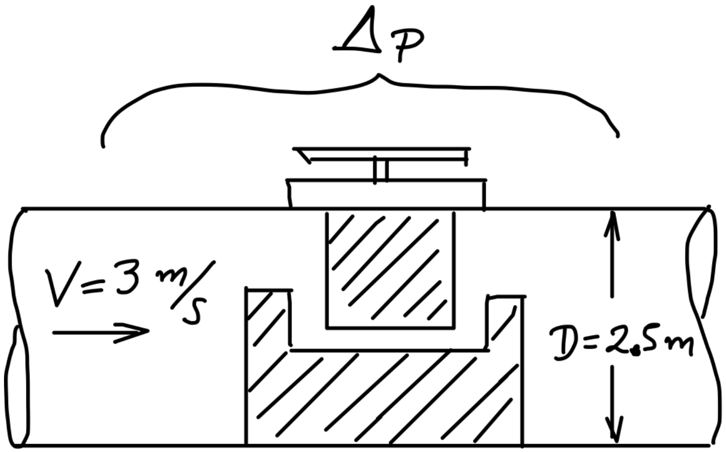
So, velocities in the model will be 20 times higher to satisfy dynamic similarity.

If water were used for the model:

$$\frac{V_m}{V} = \left(\frac{\rho}{\rho_m}\right) \cdot \left(\frac{\mu_m}{\mu}\right) \cdot \left(\frac{W}{W_m}\right)$$

$$= \frac{1.23\ \mathrm{kg/m^3}}{998\ \mathrm{kg/m^3}} \times \frac{1 \times 10^{-3}\ \mathrm{N \cdot s/m^2}}{1.82 \times 10^{-5}\ \mathrm{N \cdot s/m^2}} \times \frac{20}{1} = 1.35$$

Example: pressure drop across a valve



Alaska oil pipeline

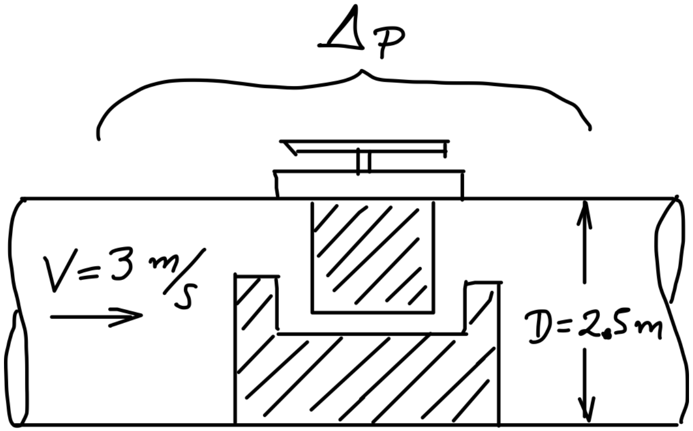
$\rho_{oil} = 930\ kg/m^3$

$\mu_{oil} = 0.48\ N \cdot s/m^2$

Use a 1:20 model with water as working fluid

– What is the flowrate through the model?

– What is the pressure drop in the prototype if the measured pressure drop in the model was $\Delta p_m = 15\ N/m^2$



$$\Delta p = f\Big(\underbrace{D, \overbrace{l_1, l_2, \ldots, l_n}^{\text{geometry of valve}}}_{\text{geometry}}, \rho, \mu, V\Big)$$

Apply π- theorem:

$$\underbrace{\frac{\Delta p}{\rho V^2}}_{\text{Euler \#}} = \phi\Big(\underbrace{\frac{l_1}{D}, \frac{l_2}{D}, \ldots, \frac{l_n}{D}}_{\text{geometry}}, \underbrace{\frac{\rho V D}{\mu}}_{\text{Reynolds \#}}\Big)$$

$$\text{Euler \#} = \frac{\text{Pressure}}{\text{inertia}} \qquad \text{Reynolds \#} = \frac{\text{Inertia}}{\text{Viscous}}$$

Design conditions:

$$\left.\begin{aligned}\frac{l_{1m}}{D_m} &= \frac{l_1}{D}\\ \frac{l_{2m}}{D_m} &= \frac{l_2}{D}\\ &\vdots\\ \frac{l_{nm}}{D_m} &= \frac{l_n}{D}\end{aligned}\right\} \Rightarrow \frac{D_m}{D} = \frac{l_{1m}}{l_1} = \frac{l_{2m}}{l_2} = \cdots = \frac{l_{nm}}{l_n} = \frac{1}{20}$$

$$n_l = \frac{1}{20} = \text{length scale}$$

Also, $\frac{\rho_m V_m D_m}{\mu_m} = \frac{\rho V D}{\mu}$

$$\Rightarrow V_m = V \cdot \left(\frac{\rho}{\rho_m}\right)\left(\frac{D}{D_m}\right)\cdot\left(\frac{\mu_m}{\mu}\right)$$

model: water

prototype: oil

$$= 3\,m/s\left(\frac{930}{999}\right)\left(\frac{20}{1}\right)\left(\frac{1.12\times10^{-3}}{0.48}\right) \Rightarrow V_m = 0.13\ m/s$$

$$Q_m = \left(\frac{\pi}{4}D_m^2\right)\cdot V_m = \frac{\pi}{4}\left(\frac{D}{20}\right)^2\cdot V_m = \frac{\pi}{4}\cdot\left(\frac{2.5\,m}{20}\right)^2 0.13\,m/s$$

$$\Rightarrow \underline{\underline{Q_m = 0.0016\ m^3/s}}$$

From the prediction equation:

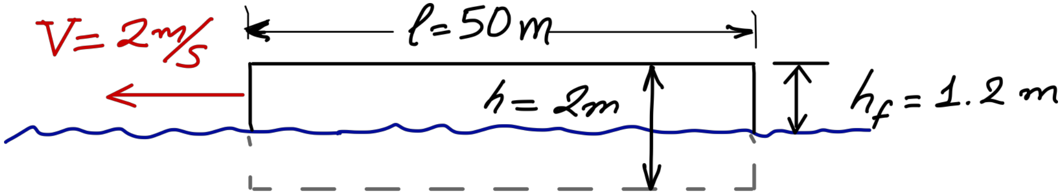
$$\frac{\Delta p_m}{\rho_m V_m^2} = \frac{\Delta p}{\rho V^2}$$

$$\Rightarrow \quad \Delta p = \Delta p_m \cdot \left(\frac{\rho}{\rho_m}\right) \cdot \left(\frac{V}{V_m}\right)^2$$

$$= 15\ \mathrm{N/m^2} \cdot \left(\frac{930}{999}\right)\left(\frac{3}{0.13}\right)^2$$

$$\Rightarrow \quad \underline{\underline{\Delta p = 7440\ \mathrm{N/m^2}}} \quad \text{or} \quad \underline{\underline{7.44\ \mathrm{kPa}}}$$

Example: flow in an open channel (gravitational effects)



To estimate the towing force of a river barge a 1:20 model is used.

a) Determine the model towing speed

b) The actual force if the model force is 6N

$$F = f(\ell, w, h, h_f, \rho, \mu, g, \sigma, V)$$

Apply the Buckingham π- theorem:

$$\frac{F}{\ell^2 \rho V^2} = \phi\left(\frac{w}{\ell}, \frac{h}{\ell}, \frac{h_f}{\ell}, \underbrace{\frac{\rho V \ell}{\mu}}_{\text{Reynolds \#}}, \underbrace{\frac{V}{\sqrt{g\ell}}}_{\text{Froude \#}}, \underbrace{\frac{\rho V^2 \ell}{\sigma}}_{\text{Weber \#}}\right)$$

$\frac{\text{inertia}}{\text{viscous}}$ $\quad$ $\frac{\text{inertia}}{\text{gravity}}$ $\quad$ $\frac{\text{inertia}}{\text{surface tension}}$

Assume that surface tension is not very important:

$$\frac{F}{l^2 \rho V^2} = \phi\left(\frac{w}{l}, \frac{h}{l}, \frac{h_f}{l}, \frac{\rho V l}{\mu}, \frac{V}{\sqrt{gl}}\right)$$

Design conditions:

$$\left.\begin{aligned}
\frac{w_m}{l_m} &= \frac{w}{l} \\
\frac{h_m}{l_m} &= \frac{h}{l} \\
\frac{h_{fm}}{l_m} &= \frac{h_f}{l}
\end{aligned}\right\} \text{ geometric similarity}$$

$$\left.\begin{aligned}
\frac{\rho_m V_m l_m}{\mu_m} &= \frac{\rho V l}{\mu} \qquad (1) \\
\frac{V_m}{\sqrt{g_m l_m}} &= \frac{V}{\sqrt{gl}} \qquad (2)
\end{aligned}\right\} \text{ dynamic similarity}$$

If we use the same fluid (water) in our model experiments:

(1) $\Rightarrow \quad V_m l_m = V l \Rightarrow \dfrac{V_m}{V} = \dfrac{l}{l_m}$

But since $g_m = g$

(2) $\Rightarrow \quad \dfrac{V_m}{\sqrt{l_m}} = \dfrac{V}{\sqrt{l}} \Rightarrow \dfrac{V_m}{V} = \sqrt{\dfrac{l_m}{l}}$

- Cannot satisfy unless $l_m = l$ (meaningless)
- <u>Distorted</u> model

Gravitational effects are the most important

→ Require Froude number similarity

$$\therefore \quad \frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}} \Rightarrow V_m = V \cdot \sqrt{\frac{\ell_m}{\ell}} = 2\,m/s \cdot \sqrt{\frac{1}{20}}$$

$$\Rightarrow \quad \underline{\underline{V_m = 0.447\ m/s}}$$

From the prediction equation:

$$\frac{F_m}{\ell_m^2 \cdot \rho_m V_m^2} = \frac{F}{\ell^2 \rho V^2} \Rightarrow F = F_m \cdot \left(\frac{\ell}{\ell_m}\right)^2 \left(\frac{V}{V_m}\right)^2$$

$$\Rightarrow F = 6N \cdot \left(\frac{20}{1}\right)^2 \cdot \left(\frac{2}{0.447}\right)^2 \Rightarrow \underline{\underline{F = 48'000\,N}}$$